Spiral cylindrique avec courbes terminales : deux arcs de cercle

Développement excentrique et anisochronisme en position horizontale

Déformations planes

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $ép = 0.09\,mm$ $ha = 0.334\,mm$ $S = 0.03\,mm^2$ $R_0 = 5\,mm$

Elinvar $\rho_s = 8\times10^3\,m^{-3}\cdot kg$ $E = 1.7\times10^{11}\,Pa$ $G = 6.538\times10^{10}\,Pa$

Partie cylindrique $n_s := 10.15$ $\psi_0 := n_s\cdot 360\cdot deg$ $\psi_0 = 3.654\times10^3\,deg$ $L := R_0\cdot\psi_0$ $L = 318.872\,mm$

$r_s(\alpha) := R_0$ $s(\alpha) := R_0\cdot(\alpha-\pi)$ $x_{0s}(\alpha) := R_0\cdot\cos(\alpha)$ $y_{0s}(\alpha) := R_0\cdot\sin(\alpha)$

Courbe terminale externe

$r_{t1} := 0.8$ $r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}{}^2\cdot(1-r_{t1})^2, r_{t1}\right]\cdot R_0$ $r_{t1} = 0.832\,R_0$

$r_{t2} := 2\cdot r_{t1} - R_0$ $r_{t2} = 0.665\,R_0$ $\beta_0 := \arctan\left[\dfrac{\pi\cdot r_{t1}}{2\cdot(R_0-r_{t1})}\right]$ $\beta_0 = 82.695\,deg$ $l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$

$x_{0t1}(\alpha_t) := -R_0 + r_{t1}\cdot(1+\cos(\alpha_t))$ $y_{0t1}(\alpha_t) := r_{t1}\cdot\sin(\alpha_t)$

$x_{0t2}(\beta_t) := r_{t2}\cdot\cos(\beta_t)$ $y_{0t2}(\beta_t) := r_{t2}\cdot\sin(\beta_t)$

Courbe terminale interne $\alpha_B := mod(\psi_0+\pi, 2\cdot\pi)$ $\alpha_B = 234\,deg$ $L_t := 2\cdot l_t + L$

$$x_{0t'1}(\alpha_{t'}) := (R_0 - r_{t1} + r_{t1}\cdot\cos(\alpha_{t'}))\cdot\cos(\alpha_B) - r_{t1}\cdot\sin(\alpha_{t'})\cdot\sin(\alpha_B)$$
$$y_{0t'1}(\alpha_{t'}) := (R_0 - r_{t1} + r_{t1}\cdot\cos(\alpha_{t'}))\cdot\sin(\alpha_B) + r_{t1}\cdot\sin(\alpha_{t'})\cdot\cos(\alpha_B)$$
$$x_{0t'2}(\beta_{t'}) := r_{t2}\cdot\cos(\beta_{t'})\cdot\cos(\alpha_B+\pi) - r_{t2}\cdot\sin(\beta_{t'})\cdot\sin(\alpha_B+\pi)$$
$$y_{0t'2}(\beta_{t'}) := r_{t2}\cdot\cos(\beta_{t'})\cdot\sin(\alpha_B+\pi) + r_{t2}\cdot\sin(\beta_{t'})\cdot\cos(\alpha_B+\pi)$$

Position du piton $r_P := r_{t2}$ $\alpha_P := -\beta_0$ $\alpha_P = -82.695\,deg$ $x_P := x_{0t2}(\alpha_P)$ $y_P := y_{0t2}(\alpha_P)$

Position du point d'attache à la virole $r_V := r_{t2}$ $\alpha_V(\theta) := Atan(x_{0t'2}(\beta_0), y_{0t'2}(\beta_0)) + \theta$ $\alpha_V(0) = 136.695\,deg$

$x_V(\theta) := r_V\cdot\cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot\sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\theta_0 := 270\cdot deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$ $W_{f3} := W_{f\_rect}(ép, ha)$ $\sigma_{max} := \dfrac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0$ $\sigma_{max} = 113.054\,\dfrac{N}{mm^2}$

Centres de masse

Partie cylindrique $z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha)$

$\zeta_{0s} := \dfrac{R_0}{L}\cdot\displaystyle\int_{\pi}^{\psi_0+\pi} z_{0s}(\alpha)\,d\alpha$ $\xi_{0s} := Re(\zeta_{0s})$ $\eta_{0s} := Im(\zeta_{0s})$ $\xi_{0s} = -0.063\,mm$ $\eta_{0s} = -0.032\,mm$

Courbe terminale externe

$$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i \cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i \cdot y_{0t2}(\beta_t)$$

$$\zeta_{0t} := \frac{1}{l_t} \cdot \left(\int_0^{\pi} z_{0t1}(\alpha_t) \cdot r_{t1} \, d\alpha_t + \int_{-\beta_0}^{0} z_{0t2}(\beta) \cdot r_{t2} \, d\beta \right)$$

$$\xi_{0t} := Re(\zeta_{0t}) \qquad \eta_{0t} := Im(\zeta_{0t}) \qquad \xi_{0t} = 9.073 \times 10^{-10} \, mm \qquad \eta_{0t} = 1.399 \, mm$$

Courbe terminale interne

$$z_{0t'1}(\alpha_{t'}) := x_{0t'1}(\alpha_{t'}) + i \cdot y_{0t'1}(\alpha_{t'}) \qquad z_{0t'2}(\beta_{t'}) := x_{0t'2}(\beta_{t'}) + i \cdot y_{0t'2}(\beta_{t'})$$

$$\zeta_{0t'} := \frac{1}{l_t} \cdot \left(\int_0^{\pi} z_{0t'1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} z_{0t'2}(\beta) \cdot r_{t2} \, d\beta \right)$$

$$\xi_{0t'} := Re(\zeta_{0t'}) \qquad \eta_{0t'} := Im(\zeta_{0t'}) \qquad \xi_{0t'} = 1.132 \, mm \qquad \eta_{0t'} = -0.822 \, mm$$

Centre de masse du spiral

$$\zeta_s := \frac{1}{L_t} \cdot \left(L \cdot \zeta_{0s} + l_t \cdot \zeta_{0t} + l_t \cdot \zeta_{0t'} \right) \qquad \zeta_s = 0 \, mm$$

Première approximation de la déformée du spiral

Courbe terminale externe

$$\varphi_{0t2}(\beta_t) := \frac{\pi}{2} + \beta_t \qquad z_P := x_P + i \cdot y_P \qquad z_{1t2}(\theta, \beta_t) := z_P + r_{t2} \cdot \int_{-\beta_0}^{\beta_t} i \cdot e^{i \cdot \beta'_t} \cdot exp\left[i \cdot \frac{\theta}{L_t} \cdot \left[r_{t2} \cdot (\beta_0 + \beta'_t) \right] \right] d\beta'_t$$

$$z_{1t2}(\theta, \beta_t) := z_P + \frac{L_t \cdot r_{t2}}{L_t + \theta \cdot r_{t2}} \cdot \left[exp\left[i \cdot \frac{\beta_t \cdot L_t + \theta \cdot r_{t2} \cdot (\beta_0 + \beta_t)}{L_t} \right] - exp(-i \cdot \beta_0) \right] \qquad z_{1C}(\theta) := z_{1t2}(\theta, 0)$$

$$\Delta\varphi_{1C}(\theta) := \frac{\theta}{L_t} \cdot r_{t2} \cdot \beta_0 \qquad \Delta\varphi_{1C}(\theta_0) = 3.653 \, deg$$

$$\varphi_{0t1}(\alpha_t) := \alpha_t + \frac{\pi}{2} \qquad \Delta z_{1t1}(\theta, \alpha_t) := r_{t1} \cdot \int_0^{\alpha_t} i \cdot e^{i \cdot \alpha'_t} \cdot exp\left(i \cdot \frac{\theta}{L_t} \cdot r_{t1} \cdot \alpha'_t \right) d\alpha'_t$$

$$\Delta z_{1t1}(\theta, \alpha_t) := \frac{L_t \cdot r_{t1}}{L_t + \theta \cdot r_{t1}} \cdot \left(exp\left(i \cdot \alpha_t \cdot \frac{L_t + \theta \cdot r_{t1}}{L_t} \right) - 1 \right) \qquad z_{1t1}(\theta, \alpha_t) := z_{1C}(\theta) + \Delta z_{1t1}(\theta, \alpha_t) \cdot e^{i \cdot (\Delta\varphi_{1C}(\theta))}$$

Partie cylindrique

$$\varphi_0(\alpha) := \alpha + \frac{\pi}{2} \qquad \Delta z_{1s}(\theta, \alpha) := R_0 \cdot \int_{\pi}^{\alpha} i \cdot exp(i \cdot \alpha') \cdot exp\left(i \cdot \theta \cdot R_0 \cdot \frac{\alpha' - \pi}{L_t} \right) d\alpha'$$

$$\Delta z_{1s}(\theta, \alpha) := \frac{R_0 \cdot L_t}{L_t + \theta \cdot R_0} \cdot \left(exp\left(-i \cdot \frac{-\alpha \cdot L_t - \theta \cdot R_0 \cdot \alpha + \theta \cdot R_0 \cdot \pi}{L_t} \right) + 1 \right) \qquad z_{1A}(\theta) := z_{1t1}(\theta, \pi)$$

$$\Delta\varphi_{1A}(\theta) := \theta \cdot \frac{l_t}{L_t} \qquad \Delta\varphi_{1A}(\theta_0) = 13.608 \, deg \qquad z_{1s}(\theta, \alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta, \alpha) \cdot e^{i \cdot \Delta\varphi_{1A}(\theta)}$$

Courbe terminale interne

$$z_{1t'1}(\theta, \alpha_{t'}) := r_{t1} \cdot \int_0^{\alpha_{t'}} i \cdot exp(i \cdot \alpha'_{t'}) \cdot exp\left(i \cdot \theta \cdot \frac{r_{t1}}{L_t} \cdot \alpha'_{t'}\right) d\alpha'_{t'}$$

$$\Delta z_{1t'1}(\theta, \alpha_{t'}) := \frac{r_{t1} \cdot L_t}{\theta \cdot r_{t1} + L_t} \cdot \left(exp\left(i \cdot \alpha_{t'} \cdot \frac{\theta \cdot r_{t1} + L_t}{L_t}\right) - 1\right) \qquad z_{1B}(\theta) := z_{1s}(\theta, \psi_0 + \pi) \qquad \alpha_B = 234\, deg$$

$$\alpha_{1B}(\theta) := Atan(Re(z_{1B}(\theta)), Im(z_{1B}(\theta))) \qquad z_{1t'1}(\theta, \alpha_{t'}) := z_{1B}(\theta) + \Delta z_{1t'1}(\theta, \alpha_{t'}) \cdot e^{i \cdot \alpha_{1B}(\theta)}$$

$$\Delta z_{1t'2}(\theta, \beta_{t'}) := r_{t2} \cdot \int_0^{\beta_{t'}} i \cdot e^{i \cdot \beta'_{t'}} \cdot exp\left[i \cdot \frac{\theta}{L_t} \cdot (r_{t2} \cdot \beta'_{t'})\right] d\beta'_{t'} \qquad z_{1C'}(\theta) := z_{1t'1}(\theta, \pi)$$

$$\Delta\varphi_{1C'}(\theta) := \alpha_{1B}(\theta) + \frac{\theta}{L_t} \cdot r_{t1} \cdot \pi + \pi \qquad z_{1t'2}(\theta, x) := z_{1C'}(\theta) + \Delta z_{1t'2}(\theta, x) \cdot e^{i \cdot \Delta\varphi_{1C'}(\theta)}$$

Graphe de la déformation

Forme naturelle

$$n_t := 201 \qquad j := 0 .. n_t - 1 \qquad \Delta\alpha_t := \frac{\pi}{n_t - 1} \qquad \alpha_{t_j} := j \cdot \Delta\alpha_t \qquad x_{t1_j} := x_{0t1}(\alpha_{t_j}) \qquad y_{t1_j} := y_{0t1}(\alpha_{t_j})$$

$$\Delta\beta_t := \frac{\beta_0}{n_t - 1} \qquad \beta_{t_j} := j \cdot \Delta\beta_t - \beta_0 \qquad x_{t2_j} := x_{0t2}(\beta_{t_j}) \qquad y_{t2_j} := y_{0t2}(\beta_{t_j})$$

$$x_t := pile(x_{t2}, x_{t1}) \qquad y_t := pile(y_{t2}, y_{t1})$$

$$n := 50 \cdot partentière(n_s) + 1 \qquad i := 0 .. n - 1 \qquad \Delta\alpha := \frac{\psi_0}{n - 1} \qquad \alpha_i := \pi + i \cdot \Delta\alpha$$

$$x_{s_i} := x_{0s}(\alpha_i) \qquad y_{s_i} := y_{0s}(\alpha_i) \qquad x_0 := pile(x_t, x_s) \qquad y_0 := pile(y_t, y_s)$$

$$\alpha_{t'_j} := j \cdot \Delta\alpha_t \qquad x_{t'1_j} := x_{0t'1}(\alpha_{t'_j}) \qquad y_{t'1_j} := y_{0t'1}(\alpha_{t'_j}) \qquad x_0 := pile(x_0, x_{t'1}) \qquad y_0 := pile(y_0, y_{t'1})$$

$$\beta_{t'_j} := j \cdot \Delta\beta_t \qquad x_{t'2_j} := x_{0t'2}(\beta_{t'_j}) \qquad y_{t'2_j} := y_{0t'2}(\beta_{t'_j}) \qquad x_0 := pile(x_0, x_{t'2}) \qquad y_0 := pile(y_0, y_{t'2})$$

$$r_0 := \overrightarrow{\sqrt{x_0^2 + y_0^2}} \qquad \beta_s := \overrightarrow{Atan(x_0, y_0)}$$

Déformée

$$z_{td2} := \overrightarrow{z_{1t2}(\theta_0, \beta_t)} \qquad z_d := z_{td2} \qquad z_{td1} := \overrightarrow{z_{1t1}(\theta_0, \alpha_t)} \qquad z_d := pile(z_{td2}, z_{td1})$$

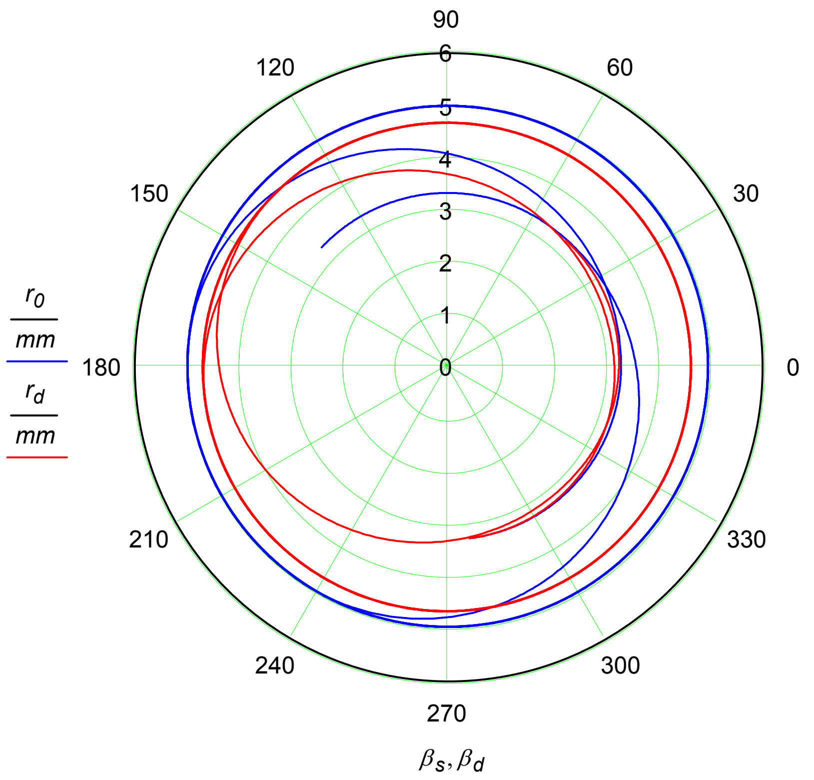
$$z_{sd} := \overrightarrow{z_{1s}(\theta_0, \alpha)} \qquad z_d := pile(z_d, z_{sd})$$

$$z_{t'd1} := \overrightarrow{z_{1t'1}(\theta_0, \alpha_{t'})} \qquad z_d := pile(z_d, z_{t'd1}) \qquad z_{t'd2} := \overrightarrow{z_{1t'2}(\theta_0, \beta_{t'})} \qquad z_d := pile(z_d, z_{t'd2})$$

$$n_{pt} := dernier(z_d) \qquad x_d := Re(z_d) \qquad y_d := Im(z_d) \qquad r_d := \overrightarrow{|z_d|} \qquad r_{d_{n_{pt}}} = 3.314\, mm$$

$$\beta_d := \overrightarrow{Atan(x_d, y_d)} \qquad \beta_{d_0} = 277.305\, deg \qquad \beta_{d_{n_{pt}}} = 46.659\, deg$$

$$mod(\alpha_V(\theta_0), 2 \cdot \pi) = 46.695\, deg$$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 3.324\,mm$

$r_V = 3.324\,mm$

$\alpha_V(0) = 136.695\,deg$

$x_V(\theta_0) = 2.28\,mm$

$y_V(\theta_0) = 2.419\,mm$

$\Delta x_V := x_{d_{npt}} - x_V(\theta_0)$

$\Delta x_V = -5.476 \times 10^{-3}\,mm$

$\Delta y_V := y_{d_{npt}} - y_V(\theta_0)$

$\Delta y_V = -8.818 \times 10^{-3}\,mm$

Déplacement de la virole libre

Contribution de la partie cylindrique du spiral

$s_s(\alpha) := R_0\cdot(\alpha - \pi) + l_t$ $\qquad$ $f_s(\theta,\alpha) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_s(\theta) := \frac{R_0}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot f_s(\theta,\alpha)\,d\alpha \qquad \Delta_s(\theta_0) = 0.101 + 0.311i\,mm$$

Approximation $\quad OA := R_0\cdot e^{i\cdot\pi} \quad OB := R_0\cdot e^{i\cdot(\pi+\psi_0)} \quad f'_s(\theta,\alpha) := \frac{-\theta^2}{L_t}\cdot R_0\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\left[\left(i\cdot f_s(\theta,\pi) - f'_s(\theta,\pi)\right)\cdot OA + \left(-i\cdot f_s(\theta,\pi+\psi_0) + f'_s(\theta,\pi+\psi_0)\right)\cdot OB\right]$$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\theta\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(-OA + e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right) + \frac{R_0^{\,2}}{L_t^{\,2}}\cdot\theta^2\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(OA - e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right)$$

$\Delta_{as}(\theta_0) = 0.1 + 0.309i\,mm$

Contribution de la courbe terminale externe

$s_{t2}(\beta_t) := r_{t2}\cdot(\beta_0 + \beta_t)$ $\qquad$ $s_{t1}(\alpha_t) := (r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t)$

$$\Delta_{t2}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t2}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t2}(\beta_t)\right)d\beta_t \qquad \Delta_{t2}(\theta_0) = 0.122 + 0.148i\,mm$$

$$\Delta_{t1}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t1}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(s_{t1}(\alpha_t)\right)\right]d\alpha_t \qquad \Delta_{t1}(\theta_0) = -0.408 - 0.209i\,mm$$

$$\Delta_t(\theta) := \Delta_{t1}(\theta) + \Delta_{t2}(\theta) \qquad \Delta_t(\theta_0) = -0.285 - 0.061i\,mm$$

Approximations

$$f_{t2}(\theta,\beta_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right) \qquad f'_{t2}(\theta,\beta_t) := \frac{-\theta^2}{L_t}\cdot r_{t2}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right)$$

$$\mathbf{Og_{12}} := \frac{r_{t2}}{l_t}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\,d\beta_t \qquad \mathbf{Og_{22}} := \frac{2\cdot r_{t2}}{l_t^2}\cdot\int_{-\beta_0}^{0} r_{t2}\cdot\beta_t\cdot z_{0t2}(\beta_t)\,d\beta_t$$

$$\Delta_{at2}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t2}(\theta,0)\cdot\mathbf{Og_{12}} + f'_{t2}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_{t2}}\cdot\mathbf{Og_{22}}\right) \qquad \Delta_{at2}(\theta_0) = 0.123 + 0.148i\,mm$$

$$f_{t1}(\theta,\alpha_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right) \qquad f'_{t1}(\theta,\alpha_t) := \frac{-\theta^2}{L_t}\cdot r_{t1}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right)$$

$$\mathbf{Og_{11}} := \frac{r_{t1}}{l_t}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\,d\alpha_t \qquad \mathbf{Og_{21}} := \frac{2\cdot r_{t1}}{l_t^2}\cdot\int_{0}^{\pi} r_{t1}\cdot\alpha_t\cdot z_{0t1}(\alpha_t)\,d\alpha_t$$

$$\Delta_{at1}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t1}(\theta,0)\cdot\mathbf{Og_{11}} + f'_{t1}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_{t1}}\cdot\mathbf{Og_{21}}\right) \qquad \Delta_{at1}(\theta_0) = -0.41 - 0.212i\,mm$$

$$\mathbf{Og_1} := \mathbf{Og_{11}} + \mathbf{Og_{12}} \qquad \mathbf{Og_1} = 9.073\times10^{-10} + 1.399i\,mm \qquad \xi_{0t} = 9.073\times10^{-10}\,mm \qquad \eta_{0t} = 1.399\,mm$$

$$\Delta_{at}(\theta) := \Delta_{at1}(\theta) + \Delta_{at2}(\theta) \qquad \Delta_{at}(\theta_0) = -0.287 - 0.064i\,mm$$

Contribution de la courbe terminale interne $\quad s_{t'1}(\alpha_{t'}) := r_{t1}\cdot\alpha_{t'} + L + l_t \qquad s_{t'2}(\beta_{t'}) := s_{t'1}(\pi) + r_{t2}\cdot\beta_{t'}$

$$\Delta_{t'1}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_{0}^{\pi} z_{0t'1}(\alpha_{t'})\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t'1}(\alpha_{t'})\right)\cdot r_{t1}\,d\alpha_{t'} \qquad \Delta_{t'1}(\theta_0) = 0.207 - 0.409i\,mm$$

$$\Delta_{t'2}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_{0}^{\beta_0} z_{0t'2}(\beta_{t'})\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t'2}(\beta_{t'})\right)\cdot r_{t2}\,d\beta_{t'} \qquad \Delta_{t'2}(\theta_0) = -0.012 + 0.192i\,mm$$

$$\Delta_{t'}(\theta) := \Delta_{t'1}(\theta) + \Delta_{t'2}(\theta) \qquad \Delta_{t'}(\theta_0) = 0.195 - 0.217i\,mm$$

Approximations

$$f_{t'1}(\theta,\alpha_{t'}) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'1}(\alpha_{t'})}{L_t}\right) \qquad f'_{t'1}(\theta,\alpha_{t'}) := \frac{-\theta^2}{L_t}\cdot r_{t1}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'1}(\alpha_{t'})}{L_t}\right)$$

$$\mathbf{Og'_{11}} := \frac{r_{t1}}{l_t}\cdot\int_{0}^{\pi} z_{0t'1}(\alpha_{t'})\,d\alpha_{t'} \qquad \mathbf{Og'_{21}} := \frac{2\cdot r_{t1}}{l_t^2}\cdot\int_{0}^{\pi} r_{t1}\cdot\alpha_{t'}\cdot z_{0t'1}(\alpha_{t'})\,d\alpha_{t'}$$

$$\Delta_{at'1}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'1}(\theta,0)\cdot\mathbf{Og'_{11}} + f'_{t'1}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_{t1}}\cdot\mathbf{Og'_{21}}\right) \qquad \Delta_{at'1}(\theta_0) = 0.21 - 0.409i\,mm$$

$$f_{t'2}(\theta,\beta_{t'}) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'2}(\beta_{t'})}{L_t}\right) \qquad f'_{t'2}(\theta,\beta_{t'}) := \frac{-\theta^2}{L_t}\cdot r_{t2}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'2}(\beta_{t'})}{L_t}\right)$$

$$Og'_{12} := \frac{r_{t2}}{l_t}\cdot\int_0^{\beta_0} z_{0t'2}(\beta_{t'})\,d\beta_{t'} \qquad Og'_{22} := \frac{2\cdot r_{t2}}{l_t^2}\cdot\int_0^{\beta_0} r_{t2}\cdot\beta_{t'}\cdot z_{0t'2}(\beta_{t'})\,d\beta_{t'}$$

$$\Delta_{at'2}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'2}(\theta,0)\cdot Og'_{12} + f'_{t'2}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_{t2}}\cdot Og'_{22}\right) \qquad \Delta_{at'2}(\theta_0) = -0.012 + 0.192i\,mm$$

$$Og'_1 := Og'_{11} + Og'_{12} \qquad Og'_1 = 1.132 - 0.822i\,mm \qquad \xi_{0t'} = 1.132\,mm \qquad \eta_{0t'} = -0.822\,mm$$

$$\Delta_{at'}(\theta) := \Delta_{at'1}(\theta) + \Delta_{at'2}(\theta) \qquad \Delta_{at'}(\theta_0) = 0.198 - 0.217i\,mm$$

Contribution du spiral entier

$$\Delta_1(\theta) := \Delta_t(\theta) + \Delta_s(\theta) + \Delta_{t'}(\theta) \qquad \Delta_1(\theta_0) = 0.011 + 0.033i\,mm$$

$$u_1(\theta) := Re(\Delta_1(\theta)) \qquad v_1(\theta) := Im(\Delta_1(\theta)) \qquad u_1(\theta_0) = 0.011\,mm \qquad v_1(\theta_0) = 0.033\,mm$$

Approximation

$$\Delta_a(\theta) := \Delta_{at}(\theta) + \Delta_{as}(\theta) + \Delta_{at'}(\theta) \qquad \Delta_a(\theta_0) = 0.011 + 0.028i\,mm$$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_\pi^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_0^\pi x_{0t1}(\alpha_t)^2\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^0 x_{0t2}(\beta_t)^2\cdot r_{t2}\,d\beta_t\right)$$

$$p2_{0s} := p2_{0s} + \frac{1}{L_t}\cdot\left(\int_0^\pi x_{0t'1}(\alpha_{t'})^2\cdot r_{t1}\,d\alpha_{t'} + \int_0^{\beta_0} x_{0t'2}(\beta_{t'})^2\cdot r_{t2}\,d\beta_{t'}\right) \qquad p2_{0s} = 12.036\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left[\int_\pi^{\pi+\psi_0} y_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_0^\pi y_{0t1}(\alpha_t)^2\cdot r_{t1}\,d\alpha_t + \left(\int_{-\beta_0}^0 y_{0t2}(\beta_t)^2\cdot r_{t2}\,d\beta_t\right)\right]$$

$$q2_{0s} := q2_{0s} + \frac{1}{L_t}\cdot\left(\int_0^\pi y_{0t'1}(\alpha_{t'})^2\cdot r_{t1}\,d\alpha_{t'} + \int_0^{\beta_0} y_{0t'2}(\beta_{t'})^2\cdot r_{t2}\,d\beta_{t'}\right) \qquad q2_{0s} = 12.072\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_\pi^{\pi+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot R_0\,d\alpha + \int_0^\pi x_{0t1}(\alpha_t)\cdot y_{0t1}(\alpha_t)\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^0 x_{0t2}(\beta_t)\cdot y_{0t2}(\beta_t)\cdot r_{t2}\,d\beta_t\right)$$

$$k_{0s} := k_{0s} + \frac{1}{L_t}\cdot\left(\int_0^\pi x_{0t'1}(\alpha_{t'})\cdot y_{0t'1}(\alpha_{t'})\cdot r_{t1}\,d\alpha_{t'} + \int_0^{\beta_0} x_{0t'2}(\beta_{t'})\cdot y_{0t'2}(\beta_{t'})\cdot r_{t2}\,d\beta_{t'}\right) \qquad k_{0s} = -0.025\,mm^2$$

$$S_0 := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s}\end{pmatrix} \qquad R'(\theta) := S_0^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} 9.614\times10^{-6} \\ 2.985\times10^{-5}\end{pmatrix} N$$

$$\left|R'(\theta_0)\right| = 3.136\times10^{-5}\,N$$

Approximations

$$\sigma 2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0}\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\,d\alpha+\int_0^{\pi}\left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_{t1}\,d\alpha_t+\int_{-\beta_0}^{0}\left(\left|z_{0t2}(\beta_t)\right|\right)^2\cdot r_{t2}\,d\beta_t\right]$$

$$\sigma 2 := \sigma 2+\frac{1}{L_t}\cdot\left[\int_0^{\pi}\left(\left|z_{0t'1}(\alpha_{t'})\right|\right)^2\cdot r_{t1}\,d\alpha_{t'}+\int_0^{\beta_0}\left(\left|z_{0t'2}(\beta_{t'})\right|\right)^2\cdot r_{t2}\,d\beta_{t'}\right]$$

$\sigma 2 = 24.108\,mm^2$ $\quad R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma 2}\cdot\begin{pmatrix}u_1(\theta)\\ v_1(\theta)\end{pmatrix}$ $\quad R'(\theta_0) = \begin{pmatrix}9.691\times 10^{-6}\\ 2.983\times 10^{-5}\end{pmatrix}N$

$\left|R'(\theta_0)\right| = 3.136\times 10^{-5}\,N$

Perturbation de période - spiral non déformé en position de repos

$X(\theta) := \frac{\left(\left|\Delta_1(\theta)\right|\right)^2}{\sigma 2}$ $\quad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$ $\quad Delta(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$

$\mu(\theta_0) := -86400\cdot Delta(\theta_0)$ $\quad \boxed{\mu(\theta_0) = 0.152}$ $\quad \boxed{\mu(180\cdot deg) = 0.243}$

$X(\theta) := \frac{\left(\left|\Delta_a(\theta)\right|\right)^2}{\sigma 2}$ $\quad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$ $\quad \delta_a(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$

$\mu_a(\theta_0) := -86400\cdot\delta_a(\theta_0)$ $\quad \boxed{\mu_a(\theta_0) = 0.184}$ $\quad \boxed{\mu_a(180\cdot deg) = 0.249}$

$\theta_m := 180\cdot deg, 190\cdot deg\,..\,360\cdot deg$